Wednesday, September 13 **Fall 2023**

Section 1: Counting and Definitions of Probability

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Based section template and practice problems by Rachel Li and Ginnie Ma '23

1. I am interested in five subjects (statistics, math, computer science, chemistry, and biology) and am deciding between 100, 102, 110, and 111 in each department (e.g., my options are Stat 100, Stat 102, Stat 110, Stat 111, Math 100, Math 102, etc.). Assume unless stated otherwise that each class (or set of classes) is equally likely to be chosen if possible.

1. **How many different schedules of 4 courses can I take?**

2. **I want to send my friend a text of my 4-class schedule. My text will look be a list like "Chemistry 101, Stat 110, Biology 111, Math 101", which is considered a different text from "Stat 110, Chemistry 101, Biology 111, Math 101". How many different such texts could I send?**

3. **If I take 5 courses, what is the probability that I take exactly 1 course from each department?**

4. **My advisor wants to know that I have backup plans, so they ask me to send 2 schedules with 4 courses each, where the schedules can have no overlap. How many such sets of two schedules can I send my advisor?**

5. **After I finalize my schedule, my advisor now wants to know how many classes I'm taking in each subject, so I send them a list that follows the following format of the following example (with the subjects always in this order):**

   [Statistics: 2, Math: 0, Computer Science: 1, Chemistry: 1, Biology: 1].

   How many such lists can I send for a 4-class schedule?

6. **If I take 6 courses, what is the probability that there's at least 1 subject in which I am not taking any courses? Solve this problem in two ways.**

2. *(Problem selection and solution taken from Rachel Li and Ginnie Ma's notes)* Hockey Stick Identity: show using a story proof that

$$\sum_{j=k}^{n} \binom{j}{k} = \binom{n+1}{k+1}.$$

Hint: Imagine arranging a group of people by age, and then think about the oldest person in a chosen subgroup.

3. Use problem 1.4 to construct a story proof for the following identity (assuming $n \geq 2k$). Recall that in problem 1.4, my advisor asked me to make 2 (non-overlapping) schedules with 4 courses each.

$$\binom{n}{2k}\binom{2k}{k} = \binom{n}{k}\binom{n-k}{k}.$$